RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, MARCH 2022

FIRST YEAR [BATCH 2021-24]

Date : 08/03/2022 **MATHEMATICS**

Time : 11am-1pm **Paper** : MTMA CC1 Full Marks : 50

Group A

Answer all the questions, maximum one can score 25.

1. Show that a non-abelian group must contain at-least six elements. [6]
2. What is the remainder when 7^{491} is divided by 6? [2]
3. Prove or disprove the following statement:
   There exist a non-abelian group of order 22 which do not have any subgroup of order 11. [5]
4. If a and b are distinct group elements, prove that either $a^2 \neq b^2$ or $a^3 \neq b^3$. [5]
5. Let G be a group such that $600 < |G| < 1100$. Suppose G has subgroups of order 56 and 63. Find the order of G. [4]
6. Suppose a and b are group elements such that $o(a) = 4, o(b) = 2$ and $a^3b = ba$.
   Find $o(ab)$. [4]
7. Compute: $((2\ 4\ 9)(3\ 1\ 7\ 8)^{-1})^{-1}$ in S_{11}. [4]

Group B

Answer all the questions, maximum one can score 25.

8. (a) Prove that $1 + \sqrt{2} + \sqrt{3}$ is not a rational number. [4]
   (b) Suppose $A \subseteq \mathbb{R}$ is bounded.
   If $\sup A = \lambda$ and $\lambda \notin A$, show that λ is a limit point of A. [2]
   (c) If $A \subseteq \mathbb{R}$ is uncountable, show that A has a limit point. [3]
9. (a) Find all limit points of $A = \left\{n + \sqrt{2} : n \in \mathbb{N}\right\}$. [2]
   (b) Find two sets A, B in $\mathbb{R}$ such that $(A \cup B)^\circ \neq A^\circ \cup B^\circ$. [2]
   (c) Suppose F is a finite subset of $\mathbb{Q}$. Prove that $\mathbb{Q} - F$ is dense in $\mathbb{R}$. [2]
10. (a) If $A \subseteq \mathbb{R}$ and r is a limit point of A, prove that there is a sequence $\{x_n\} \subseteq A$ such that $\{x_n\}$ converges to r. [3]
    (b) Suppose $\{x_n\}, \{y_n\}$ are two sequences in $\mathbb{R}$ such that $\{x_n y_n\}$ is convergent. Is at least one of $\{x_n\}$ *or* $\{y_n\}$ necessarily bounded? Justify your answer. [3]
    (c) Find the $\limsup x_n$ and $\liminf x_n$ where $x_n = (-1)^n \left(1 + \frac{1}{n}\right)\ \forall n \in \mathbb{N}$. [2]
11. (a) Let $f : \mathbb{R} \to \mathbb{R}$ be such that
    $$f(x) = \begin{cases} x, & x \in \mathbb{Q} \\ 2 - x, & x \in \mathbb{R} - \mathbb{Q} \end{cases}$$
    Show that $\lim\limits_{x \to 1} f(x) = 1$. [3]
    (b) Which of the following sets is countable? Justify your answer.
    i. $(\mathbb{R} - \mathbb{Q}) - \left\{n + \sqrt{3} : n \in \mathbb{N}\right\}$ [2]
    ii. $\mathbb{N} \times \mathbb{N}$. [2]

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